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$$\therefore 4\xi\eta\frac{\partial^2 u}{\partial\xi\,\partial\eta}=\left(\frac{\partial}{\partial x}+y\frac{\partial}{\partial y}\right)\left(\frac{\partial}{\partial x}-y\frac{\partial}{\partial y}\right)u=\frac{\partial^2 u}{\partial x^2}-y\frac{\partial^2 u}{\partial x\,\partial y}+y\frac{\partial^2 u}{\partial x\,\partial y}$$

$$-y\frac{\partial u}{\partial y}-y^2\frac{\partial^2 u}{\partial y^2}=\frac{\partial^2 u}{\partial x^2}-y^2\frac{\partial^2 u}{\partial y^2}-y\frac{\partial u}{\partial y}.$$

Also solved by G. B. M. Zerr.

MECHANICS.

170. Proposed by ELISHA S. LOOMIS, Ph. D., Berea, Ohio.

Two angles of iron, A_1CD and A_1CA_3, move freely on a pivot at C. Rods B_1A_1 and B_1A_3 are attached, respectively, at A_1 and at some point A_3 so that when B_1 moves along the rod CR, which is perpendicular to A_1A_4, CD and CA_3 shall coincide in position with CE which is perpendicular to rod KR. When angle A_1CD is 135° find CA_3 in terms of CA_1. Also find the following:

a) That value of CB_1 which will require least effort exerted at B_1 to cause CA_3 to take the position CA_4.

b) That value of CB_1 which will cause B_2A_2, if produced, to pass through the point A_1.

c) As CB_1 varies in value, what is the locus of the intersection of A_1B_1 and A_2B_2? Of B_1A_3 and B_2A_4?

d) Suppose angle A_1CD to be any other angle than 135°, then find CA_3 in terms of CA_1.

(For figure see pp. 177 and 212.)

II. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let $CB_1=a$, $CA_1=r$, $CA_3=x$, $\angle A_1CA_2=\angle A_3CA_4=\beta$.

$\therefore A_1B_1=\sqrt{(a^2+r^2)}$.

d) $A_2B_2=A_1B_1=\sqrt{(a^2+r^2)}=\sqrt{(r^2+CB_2^2-2r.CB_2\sin\beta)}$.

$\therefore CB_2=r\sin\beta\pm\sqrt{(a^2+r^2\sin^2\beta)}$.

$A_4B_2=x^2+2r^2\sin^2\beta+a^2+2r\sin\beta\sqrt{(a^2+r^2\sin^2\beta)}$.

$A_3B_1=A_4B_2=x^2+a^2+2ax\sin\beta$.

$$\therefore x=\frac{r^2\sin\beta+r(a^2+r^2\sin^2\beta)}{a}.$$

When $\beta=\frac{1}{4}\pi$, $x=\dfrac{r\sqrt{2}\,[r+\sqrt{(2a^2+r^2)}]}{2a}$.

a) Let P=pull along A_3B_1, Q=resolved part of pull along A_3C.

Then $Q=P\cos CA_3B_1=P\cos\theta$

$$=\frac{P[2x+\sqrt{(2)}a}{2\sqrt{[x^2+a^2+ax\sqrt{2}]}}=\frac{Pr[r+a\sqrt{2}+\sqrt{(2a^2+r^2)}]}{[r+\sqrt{(2a^2+r^2)}]\sqrt{(a^2+r^2)}}$$

$$=\frac{Pr}{\sqrt{(a^2+r^2)}}+\frac{Pra\sqrt{2}}{[r+\sqrt{(2a^2+r^2)}]\sqrt{(a^2+r^2)}}=a \text{ minimum}.$$

Differentiating and reducing,

$$(r^3\sqrt{2}-2ar^2-2a^3)\sqrt{(2a^2+r^2)}+r^4\sqrt{2}+2a^4\sqrt{2}-4a^3r-2ar^3=0.$$

Let $r^3/a^3=y^3$.

$$\therefore (y^3\sqrt{2}-2ay^2-2)\sqrt{(2+y^2)}+y^4 2\sqrt{2}-4y-2y^3=0.$$

$\therefore y=0$ or $a=\infty$ for the minimum effort.

b) $CB_2=\dfrac{r+\sqrt{(2a^2+r^2)}}{\sqrt{2}}$, $A_2B_2=\sqrt{(a^2+r^2)}$, $A_1A_2=r\sqrt{(2-\sqrt{2})}$.

$$A_1B_2=\sqrt{[a^2+2r^2+r\sqrt{(2a^2+r^2)}]}=A_2B_2+A_1A_2=\sqrt{(a^2+r^2)}+r\sqrt{(2-\sqrt{2})}.$$

Let $a^2+r^2=y^2$.

$$\therefore \sqrt{[y^2+r^2+r\sqrt{(2y^2-r^2)}]}=y+r\sqrt{(2-\sqrt{2})}$$

$$\text{or } r(1-\sqrt{2})+2y\sqrt{(2-\sqrt{2})}=\sqrt{(2y^2-r^2)}.$$

$$\therefore y=\frac{r\sqrt{(2-\sqrt{2})}}{\sqrt{(2)}-1},\quad a=\frac{r}{\sqrt{[\sqrt{(2)}-1]}}.$$

c) $x/a-y/r=1$ is the equation to A_1B_1 ....(1).

$rx+y\sqrt{(2a^2+r^2)}-r^2\sqrt{2}-r\sqrt{2}\sqrt{(2a^2+r^2)}=0$ is equation to A_2B_2 ....(2).

From (1), $a=\dfrac{rx}{r+y}$; this in (2) gives

$$(y-r\sqrt{2})^2[2x^2+(r+y)^2]=(r+y)^2(r\sqrt{2}-x)^2 \text{ for first locus.}$$

$$\frac{2ax}{r\sqrt{2}[r+\sqrt{(2a^2+r^2)}]}+\frac{y\sqrt{2}}{r+\sqrt{(2a^2+r^2)}}=1 \text{ is equation to } A_4B_2 \text{ ....(3).}$$

$$\therefore 2ax+2ry=r\sqrt{2}[r+\sqrt{(2a^2+r^2)}] \text{ ....(4).}$$

$$[r^2-2a^2+r\sqrt{(2a^2+r^2)}]y=r[r+\sqrt{(2a^2+r^2)}]x-ar[r+\sqrt{(2a^2+r^2)}]$$

is equation to A_3B_1 ....(5).

From (4),

$$a=\frac{r^2x\sqrt{2}-2rxy\pm r\sqrt{[4x^2y^2+2r^2x^2-2\sqrt{(2)}r^3y]}}{2x^2-r^2}.$$

Substituted in (5) this gives the second locus.

171. **Proposed by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.**

Prove that the electrical capacity of an oblate ellipsoid of revolution is $\sqrt{(a^2-b^2)}/\cos^{-1}(b/a)$, where a and b are the equatorial and polar semi-diameters.

I. **Solution by M. E. GRABER, A. M., Heidelberg University, Tiffin, Ohio.**

Suppose the thickness of the shell to be $m\phi$, then the force due to the shell at any point P, external, is $4\pi\rho m\phi$. The force due to the shell is obtained from $-\dfrac{dn}{du}=\dfrac{4\pi\rho m\phi abc}{\sqrt{[(a^2+l)(b^2+l)(c^2+l)]}}$, where dn is an element of the normal at ρ.